

Quantum and Classical Gauge Symmetries¹

Kazuo Fujikawa and Hiroaki Terashima

*Department of Physics, University of Tokyo
Bunkyo-ku, Tokyo 113, Japan*

Abstract

The use of the mass term of the gauge field as a gauge fixing term, which was discussed by Zwanziger, Parrinello and Jona-Lasinio in a large mass limit, is related to the non-linear gauge by Dirac and Nambu. We have recently shown that this use of the mass term as a gauge fixing term is in fact identical to the conventional local Faddeev-Popov formula without taking a large mass limit, if one takes into account the variation of the gauge field along the entire gauge orbit. This suggests that the classical massive vector theory, for example, could be re-interpreted as a gauge invariant theory with a gauge fixing term added in suitably quantized theory. As for massive gauge particles, the Higgs mechanics, where the mass term is gauge invariant, has a more intrinsic meaning. We comment on several implications of this observation.

1 Introduction

The Faddeev-Popov formula[1] and the resulting BRST symmetry[2] provide a basis for the modern quantization of gauge theory. On the other hand, a modified quantization scheme[3][4]

$$\int \mathcal{D}A_\mu \{ \exp[-S_{YM}(A_\mu) - \int f(A_\mu)dx] / \int \mathcal{D}g \exp[-\int f(A_\mu^g)dx] \} \quad (1)$$

with, for example, $f(A_\mu) = (m^2/2)(A_\mu)^2$ has been recently analyzed in a large mass limit in connection with the analysis of Gribov-type complications[5]. This gauge fixing in the large mass limit is related to the limit $\lambda = 0$ of the non-linear gauge

$$A_\mu^2 = \lambda = \text{const.} \quad (2)$$

discussed by Dirac and Nambu[6] many years ago. Nambu used the above gauge to analyze the possible spontaneous breakdown of Lorentz symmetry. In his treatment, the limit $\lambda = 0$ is singular, and thus the present formulation is not quite convenient for an

¹Talk given at ICHEP2000, Osaka, Japan, July 2000 (to be published in the Proceedings (World Scientific, Singapore))

analysis of the possible breakdown of Lorentz symmetry. Some aspects of this non-linear gauge have been discussed in Ref[7]. The above gauge fixing (1) has also been used in lattice simulation[8].

We have recently shown[9] that the above scheme (1) is in fact identical to the conventional *local* Faddeev-Popov formula

$$\int \mathcal{D}A_\mu \delta(D^\mu \frac{\delta f(A_\nu)}{\delta A_\mu}) \det\{\delta[D^\mu \frac{\delta f(A_\nu^g)}{\delta A_\mu^g}]/\delta g\} \exp[-S_{YM}(A_\mu)] \quad (3)$$

without taking the large mass limit, if one takes into account the variation of the gauge field along the entire gauge orbit parametrized by the gauge parameter g . The above equivalence is valid only if the Gribov-type complications are ignored.

We here comment on the possible implications[10] of the above equivalence, which is established without taking the large mass limit, in a more general context of quantum gauge symmetry, namely, BRST symmetry.

2 Abelian example

We first briefly illustrate the proof[9] of the above equivalence of (1) and (3) by using an example of Abelian gauge theory, $S_0 = -(1/4) \int dx (\partial_\mu A_\nu - \partial_\nu A_\mu)^2$, for which we can work out everything explicitly. In this note we exclusively work on Euclidean theory with metric convention $g_{\mu\nu} = (1, 1, 1, 1)$. As a simple and useful example, we choose the gauge fixing function $f(A) \equiv (1/2)A_\mu A_\mu$ and

$$D_\mu(\frac{\delta f}{\delta A_\mu}) = \partial_\mu A_\mu. \quad (4)$$

Our claim above suggests the relation

$$\begin{aligned} Z &= \int \mathcal{D}A_\mu^\omega \{e^{-S_0(A_\mu^\omega) - \int dx \frac{1}{2}(A_\mu^\omega)^2} / \int \mathcal{D}h e^{-\int dx \frac{1}{2}(A_\mu^{h\omega})^2}\} \\ &= \int \mathcal{D}A_\mu^\omega \mathcal{D}B \mathcal{D}\bar{c} \mathcal{D}c e^{-S_0(A_\mu^\omega) + \int [-iB\partial_\mu A_\mu^\omega + \bar{c}(-\partial_\mu \partial_\mu)c]dx} \end{aligned} \quad (5)$$

where the variable A_μ^ω stands for the field variable obtained from A_μ by a gauge transformation parametrized by the gauge orbit parameter ω . To establish this result, we first evaluate

$$\begin{aligned} &\int \mathcal{D}h e^{-\int dx \frac{1}{2}(A_\mu^{h\omega})^2} \\ &= \int \mathcal{D}h e^{-\int dx \frac{1}{2}(A_\mu^\omega + \partial_\mu h)^2} \\ &= \int \mathcal{D}h e^{-\int dx \frac{1}{2}[(A_\mu^\omega)^2 - 2(\partial_\mu A_\mu^\omega)h + h(-\partial_\mu \partial_\mu)h]} \\ &= \int \mathcal{D}B \frac{1}{\det \sqrt{-\partial_\mu \partial_\mu}} e^{-\int dx \frac{1}{2}[(A_\mu^\omega)^2 - 2(\partial_\mu A_\mu^\omega) \frac{1}{\sqrt{-\partial_\mu \partial_\mu}} B + B^2]} \\ &= \frac{1}{\det \sqrt{-\partial_\mu \partial_\mu}} e^{-\int dx \frac{1}{2}(A_\mu^\omega)^2 + \frac{1}{2} \int \partial_\mu A_\mu^\omega \frac{1}{-\partial_\mu \partial_\mu} \partial_\nu A_\nu^\omega dx} \end{aligned} \quad (6)$$

where we defined $\sqrt{-\partial_\mu \partial_\mu} h = B$. Thus

$$\begin{aligned} Z &= \int \mathcal{D}A_\mu^\omega \{ \det \sqrt{-\partial_\mu \partial_\mu} \} e^{-S_0(A_\mu^\omega) - \frac{1}{2} \int \partial_\mu A_\mu^\omega \frac{1}{-\partial_\mu \partial_\mu} \partial_\nu A_\nu^\omega dx} \\ &= \int \mathcal{D}A_\mu^\omega \mathcal{D}B \mathcal{D}\bar{c} \mathcal{D}c e^{-S_0(A_\mu^\omega) - \frac{1}{2} \int B^2 dx + \int [-iB \frac{1}{\sqrt{-\partial_\mu \partial_\mu}} \partial_\mu A_\mu^\omega + \bar{c} \sqrt{-\partial_\mu \partial_\mu} c] dx} \end{aligned} \quad (7)$$

which was derived in Refs.[3] and [4] and is invariant under the BRST transformation

$$\begin{aligned} \delta A_\mu^\omega &= i\lambda \partial_\mu c, \quad \delta c = 0 \\ \delta \bar{c} &= \lambda B, \quad \delta B = 0 \end{aligned} \quad (8)$$

with a Grassmann parameter λ . Note the appearance of the imaginary factor i in the term $iB \frac{1}{\sqrt{-\partial_\mu \partial_\mu}} \partial_\mu A_\mu^\omega$ in (7).

We next rewrite the expression (7) as

$$\begin{aligned} &\int \mathcal{D}A_\mu^\omega \mathcal{D}B \mathcal{D}\Lambda \mathcal{D}\bar{c} \mathcal{D}c \delta \left(\frac{1}{\sqrt{-\partial_\mu \partial_\mu}} \partial_\mu A_\mu^\omega - \Lambda \right) e^{-S_0(A_\mu^\omega) - \frac{1}{2} \int (B^2 + 2i\Lambda B) dx + \int \bar{c} \sqrt{-\partial_\mu \partial_\mu} c dx} \\ &= \int \mathcal{D}A_\mu^\omega \mathcal{D}\Lambda \mathcal{D}\bar{c} \mathcal{D}c \delta \left(\frac{1}{\sqrt{-\partial_\mu \partial_\mu}} \partial_\mu A_\mu^\omega - \Lambda \right) e^{-S_0(A_\mu^\omega) - \frac{1}{2} \int \Lambda^2 dx + \int \bar{c} \sqrt{-\partial_\mu \partial_\mu} c dx}. \end{aligned} \quad (9)$$

We note that we can compensate any variation of $\delta\Lambda$ by a suitable change of gauge parameter $\delta\omega$ inside the δ -function as

$$\frac{1}{\sqrt{-\partial_\mu \partial_\mu}} \partial_\mu \partial_\mu \delta\omega = \delta\Lambda. \quad (10)$$

By a repeated application of infinitesimal gauge transformations combined with the invariance of the path integral measure under these gauge transformations, we can re-write the formula (9) as

$$\begin{aligned} &\int \mathcal{D}A_\mu^\omega \mathcal{D}\Lambda \mathcal{D}\bar{c} \mathcal{D}c \delta \left(\frac{1}{\sqrt{-\partial_\mu \partial_\mu}} \partial_\mu A_\mu^\omega \right) e^{-S_0(A_\mu^\omega) - \frac{1}{2} \int \Lambda^2 dx + \int \bar{c} \sqrt{-\partial_\mu \partial_\mu} c dx} \\ &= \int \mathcal{D}A_\mu^\omega \mathcal{D}\bar{c} \mathcal{D}c \delta \left(\frac{1}{\sqrt{-\partial_\mu \partial_\mu}} \partial_\mu A_\mu^\omega \right) e^{-S_0(A_\mu^\omega) + \int \bar{c} \sqrt{-\partial_\mu \partial_\mu} c dx} \\ &= \int \mathcal{D}A_\mu^\omega \mathcal{D}B \mathcal{D}\bar{c} \mathcal{D}c e^{-S_0(A_\mu^\omega) + \int [-iB \frac{1}{\sqrt{-\partial_\mu \partial_\mu}} \partial_\mu A_\mu^\omega + \bar{c} \sqrt{-\partial_\mu \partial_\mu} c] dx} \\ &= \int \mathcal{D}A_\mu^\omega \mathcal{D}B \mathcal{D}\bar{c} \mathcal{D}c e^{-S_0(A_\mu^\omega) + \int [-iB \partial_\mu A_\mu^\omega + \bar{c} (-\partial_\mu \partial_\mu) c] dx}. \end{aligned} \quad (11)$$

In the last stage of this equation, we re-defined the *auxiliary* variables B and \bar{c} as

$$B \rightarrow B \sqrt{-\partial_\mu \partial_\mu}, \quad \bar{c} \rightarrow \bar{c} \sqrt{-\partial_\mu \partial_\mu} \quad (12)$$

which is consistent with BRST symmetry and leaves the path integral measure invariant. We have thus established the desired result (5). We emphasize that the integral over the

entire gauge orbit, as is indicated in (10), is essential to derive a local theory (11) without taking a large mass limit[9].

It is shown that this procedure works for the non-Abelian case also[9], though the actual procedure is much more involved, if the (ill-understood) Gribov-type complications can be ignored such as in perturbative calculations.

3 Possible Implications

In the classical level, we traditionally consider

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}m^2 A_\mu A^\mu \quad (13)$$

as a Lagrangian for a massive vector theory, and

$$\mathcal{L}_{eff} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}(\partial_\mu A^\mu)^2 \quad (14)$$

as an effective Lagrangian for Maxwell theory with a Feynman-type gauge fixing term added. The physical meanings of these two Lagrangians are thus completely different.

However, the analysis in Section 2 shows that the Lagrangian (13) could in fact be interpreted as a gauge fixed Lagrangian of *massless* Maxwell field in quantized theory. To be explicit, by using (5), the Lagrangian (13) may be regarded as an effective Lagrangian in

$$\begin{aligned} Z &= \int \mathcal{D}A_\mu^\omega \{ e^{\int dx [-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}m^2 A_\mu^\omega A^{\omega\mu}]} / \int \mathcal{D}h e^{-\int dx \frac{m^2}{2} (A_\mu^{h\omega})^2} \} \\ &= \int \mathcal{D}A_\mu^\omega \mathcal{D}B \mathcal{D}\bar{c} \mathcal{D}c e^{\int dx [-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - iB\partial_\mu A_\mu^\omega + \bar{c}(-\partial_\mu \partial_\mu)c]} \end{aligned} \quad (15)$$

where we absorbed the factor m^2 into the definition of B and \bar{c} .

One can also analyze (14) by defining $f(A_\mu) \equiv \frac{1}{2}(\partial_\mu A^\mu)^2$ in the modified quantization scheme (1). The equality of (1) and (3) then gives

$$\begin{aligned} &\int \mathcal{D}A_\mu \delta(D^\mu \frac{\delta f(A_\nu)}{\delta A_\mu}) \det\{\delta[D^\mu \frac{\delta f(A_\nu^g)}{\delta A_\mu^g}]/\delta g\} \exp[-S_0(A_\mu)] \\ &= \int \mathcal{D}A_\mu \delta(\partial_\nu \partial^\nu (\partial^\mu A_\mu)) \det[\partial_\nu \partial^\nu \partial_\mu \partial^\mu] \exp[-S_0(A_\mu)] \\ &= \int \mathcal{D}A_\mu \mathcal{D}B \mathcal{D}\bar{c} \mathcal{D}c \exp\{-S_0(A_\mu) + \int dx [-iB\partial_\nu \partial^\nu (\partial^\mu A_\mu) - \bar{c}(\partial_\nu \partial^\nu \partial_\mu \partial^\mu)c]\} \end{aligned} \quad (16)$$

After the re-definition of *auxiliary* variables, $B\partial_\nu \partial^\nu \rightarrow B$, $\bar{c}\partial_\nu \partial^\nu \rightarrow \bar{c}$, which preserves BRST symmetry, (16) becomes

$$\int \mathcal{D}A_\mu \mathcal{D}B \mathcal{D}\bar{c} \mathcal{D}c \exp\{-S_0(A_\mu) + \int dx [-iB(\partial^\mu A_\mu) + \bar{c}(-\partial_\mu \partial^\mu)c]\} \quad (17)$$

which agrees with (11) and (15). We can thus assign an identical physical meaning to two classical Lagrangians (13) and (14) in suitably *quantized* theory.

Similarly, the two classical Lagrangians related to Yang-Mills fields

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c)^2 - \frac{m^2}{2}A_\mu^a A^{a\mu} \quad (18)$$

and

$$\mathcal{L}_{eff} = -\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c)^2 - \frac{1}{2}(\partial_\mu A^{a\mu})^2 \quad (19)$$

could be assigned an identical physical meaning as an effective gauge fixed Lagrangian associated with the quantum theory defined by[9]

$$\int \mathcal{D}A_\mu^a \mathcal{D}B^a \mathcal{D}\bar{c}^a \mathcal{D}c^a \exp\{-S_{YM}(A_\mu^a) + \int dx[-iB^a(\partial^\mu A_\mu^a) + \bar{c}^a(-\partial_\mu(D^\mu c)^a)]\} \quad (20)$$

which is invariant under BRST symmetry.

We have illustrated that the apparent “massive gauge field” in the classical level has no intrinsic physical meaning. It can be interpreted either as a massive (non-gauge) vector theory, or as a gauge-fixed effective Lagrangian for a massless gauge field in quantized theory. In the framework of path integral, these different *interpretations* may also be understood as a more flexible choice of the path integral measure than the classical Poisson bracket analysis suggests[10]: One choice of the measure

$$\begin{aligned} & \int d\mu \exp\left\{\int dx\left[-\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c)^2 - \frac{m^2}{2}A_\mu^a A^{a\mu}\right]\right\} \\ & \equiv \int \mathcal{D}A_\mu \frac{1}{\int \mathcal{D}g \exp\left[-\int \frac{m^2}{2}(A_\mu^{ag})^2 dx\right]} \\ & \times \exp\left\{\int dx\left[-\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c)^2 - \frac{m^2}{2}A_\mu^a A^{a\mu}\right]\right\} \end{aligned} \quad (21)$$

gives rise to a renormalizable massless gauge theory, and the other naive choice

$$\begin{aligned} & \int d\mu \exp\left\{\int dx\left[-\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c)^2 - \frac{m^2}{2}A_\mu^a A^{a\mu}\right]\right\} \\ & \equiv \int \mathcal{D}A_\mu \exp\left\{\int dx\left[-\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c)^2 - \frac{m^2}{2}A_\mu^a A^{a\mu}\right]\right\} \end{aligned} \quad (22)$$

gives rise to a non-renormalizable massive *non-gauge* theory. A somewhat analogous situation arises when one attempts to quantize the so-called anomalous gauge theory: A suitable choice of the measure with a Wess-Zumino term gives rise to a consistent quantum theory in 2-dimensions, for example[11]. From a view point of classical-quantum correspondence, one can define a classical theory uniquely starting from quantum theory by considering the limit $\hbar \rightarrow 0$, but not the other way around in general.

In the context of the present general interpretation of apparently massive classical gauge fields, the massive gauge fields generated by the Higgs mechanism are exceptional and quite different. Since all the terms including the mass term are gauge invariant, one can assign an intrinsic meaning to the massive gauge field in Higgs mechanism. In view of the well known fact that the massive non-Abelian gauge theory is inconsistent as a

quantum theory (22), it may be sensible to treat all the classical massive non-Abelian Lagrangians as a gauge fixed version of pure non-Abelian gauge theory and to restrict the massive non-Abelian gauge fields to those generated by the Higgs mechanism.

It is a long standing question if one can generate gauge fields from some *more* fundamental mechanism. To our knowledge, however, there exists no definite convincing scheme so far. On the contrary, there is a no-go theorem or several arguments against such an attempt[12]. Apart from technical details, the basic argument against the “dynamical” generation of gauge fields is that the Lorentz invariant positive definite theory cannot simply generate the negative metric states associated with the time components of massless gauge fields. In contrast, the dynamical generation of the Lagrangian of the structure

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c)^2 - \frac{m^2}{2}(A_\mu^a)^2 \quad (23)$$

does not appear to be prohibited by general arguments so far. If one considers that the induced Lagrangian such as (23) is a *classical* object which should be quantized anew, one could regard $\frac{m^2}{2}(A_\mu^a)^2$, which breaks classical gauge symmetry, as a gauge fixing term in the modified quantization scheme[3][4]. In this interpretation, one might be allowed to say that massless gauge fields are generated dynamically. Although a dynamical generation of pure gauge fields is prohibited, a *gauge fixed* Lagrangian might be allowed to be generated. (In this respect, one may recall that much of the arguments for the no-go theorem[12] would be refuted if one could generate a gauge fixed Lagrangian with the Faddeev-Popov term added.) The mass for the gauge field which has an intrinsic unambiguous physical meaning is then further induced by the spontaneous symmetry breaking of the gauge symmetry thus defined (the Higgs mechanism).

We next comment on a mechanism for generating gauge fields by the violent random fluctuation of gauge degrees of freedom at the beginning of the universe[13]; this scheme is based on the renormalization group flow starting from an initial chaotic theory. In such a scheme, it is natural to think that one is always dealing with quantum theory, and thus no room for our way of re-interpretation of the induced theory. Nevertheless, we find a possible connection in the following sense: To be precise, an example of massive Abelian gauge field in *compact* lattice gauge theory

$$\int \mathcal{D}U \frac{\mathcal{D}\Omega}{\text{vol}(\Omega)} \exp[-S_{inv}(U) - S_{mass}(U^\Omega)] \quad (24)$$

is analyzed in Ref.[13]. Here $S_{inv}(U)$ stands for the gauge invariant part of the lattice Abelian gauge field U , and $S_{mass}(U^\Omega)$ stands for the gauge non-invariant mass term with the gauge freedom Ω . In compact theory, one need not fix the gauge and instead one may take an average over the entire gauge volume of Ω . They argued that the mass term, which breaks gauge symmetry softly, disappears in the long distance limit when one integrates over the entire gauge freedom Ω . Their scheme is apparently dynamical one, in contrast to the kinematical nature of our re-interpretation. Nevertheless, the massive Abelian theory is a free theory in continuum formulation, and the disappearance of the mass term by a mere smearing over the gauge volume may suggest that the mass term in their scheme is also treated as a kind of gauge artifact, just as in our kinematical re-interpretation.

In conclusion, the equivalence of (1) and (3) allows a more flexible *quantum interpretation* of various classical Lagrangians such as massive gauge theory.

As for a recent BRST analysis of the observation in Ref.[10], see Ref.[14].

References

- [1] L.D. Faddeev and V.N. Popov, Phys. Lett. **B25**(1967)29.
- [2] C. Becchi, A. Rouet and R. Stora, Comm. Math. Phys. **42** (1975)127.
J. Zinn-Justin, Lecture Notes in Physics, **37** (Springer-Verlag, Berlin, 1975)2.
- [3] D. Zwanziger, Nucl.Phys. **B345** (1990) 461 ; **B192** (1981) 259.
- [4] G. Parrinello and G. Jona-Lasinio, Phys.Lett.**B251**(1990)175.
- [5] V.N. Gribov, Nucl. Phys. **B139**(1978)1.
- [6] Y. Nambu, Suppl. Prog. Theor. Phys. Extra Number (1968)190.
P.A.M. Dirac, Proc. Roy. Soc.(London) **A209**(1951)291.
- [7] K. Fujikawa, Phys. Rev. **D7**(1973)393.
- [8] W. Bock, M. Golterman, M. Ogilvie and Y. Shamir, hep-lat/0004017.
- [9] K. Fujikawa and H. Terashima, Nucl. Phys.**B577**(2000)405.
- [10] K. Fujikawa and H. Terashima, hep-th/0004190.
- [11] R. Jackiw and R. Rajaraman, Phys. Rev. Lett. **54** (1985) 1219; *ibid.*, **55** (1985) 224..
K. Harada and I. Tsutsui, Phys. Lett. **183B** (1987) 311.
O. Babelon, F. A. Schaposnik, and C.M. Viallet, Phys. Lett. **177B** (1986) 385.
- [12] K.M. Case and S. Gasiorowicz, Phys. Rev. **125**(1962)1055.
S. Coleman and E. Witten, Phys. Rev. Lett. **45**(1980)100.
S. Weinberg and E. Witten, Phys. Lett. **B96**(1980)59.
- [13] D. Foerster, H.B. Nielsen and M. Ninomiya, Phys. Lett. **B94**(1980)135, and references therein.
- [14] R. Banerjee and B.P. Mandal, Phys. Lett. **B488** (2000)27.